NEOCLASSICAL THEORY OF PEDESTAL FLOWS

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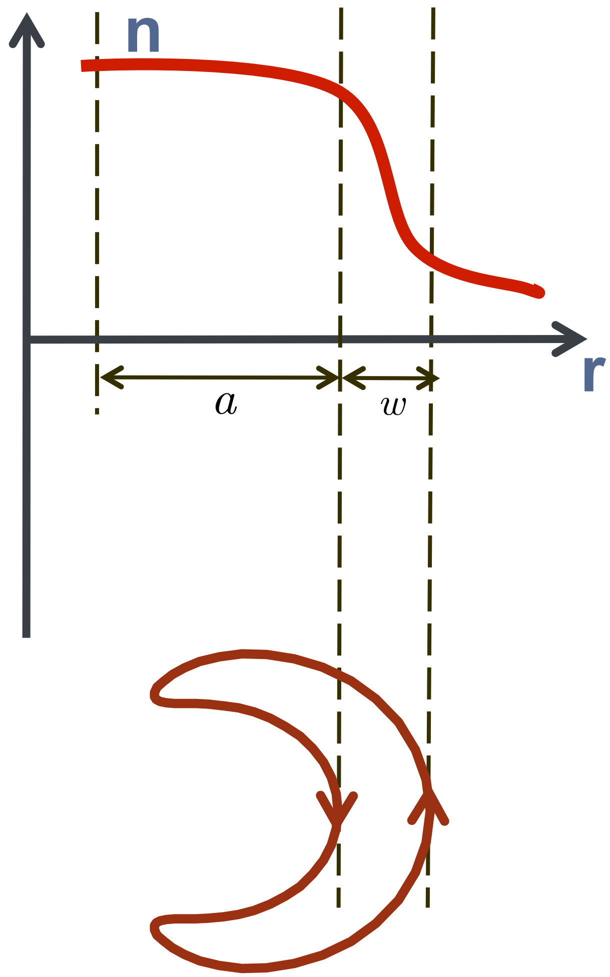
AGENDA

- Pedestal basics
- Impurity measurements in the C-Mod pedestal
- Poloidal ion flow in the presence of strong radial electric field
- Enhancement of the bootstrap current in the banana regime pedestal

PEDESTAL BASICS. MAIN ION TEMPERATURE

$\rho \ll \rho_{pol} \sim w \ll a$



$\rho_{pol} \equiv v_i Mc / ZeB_{pol}$

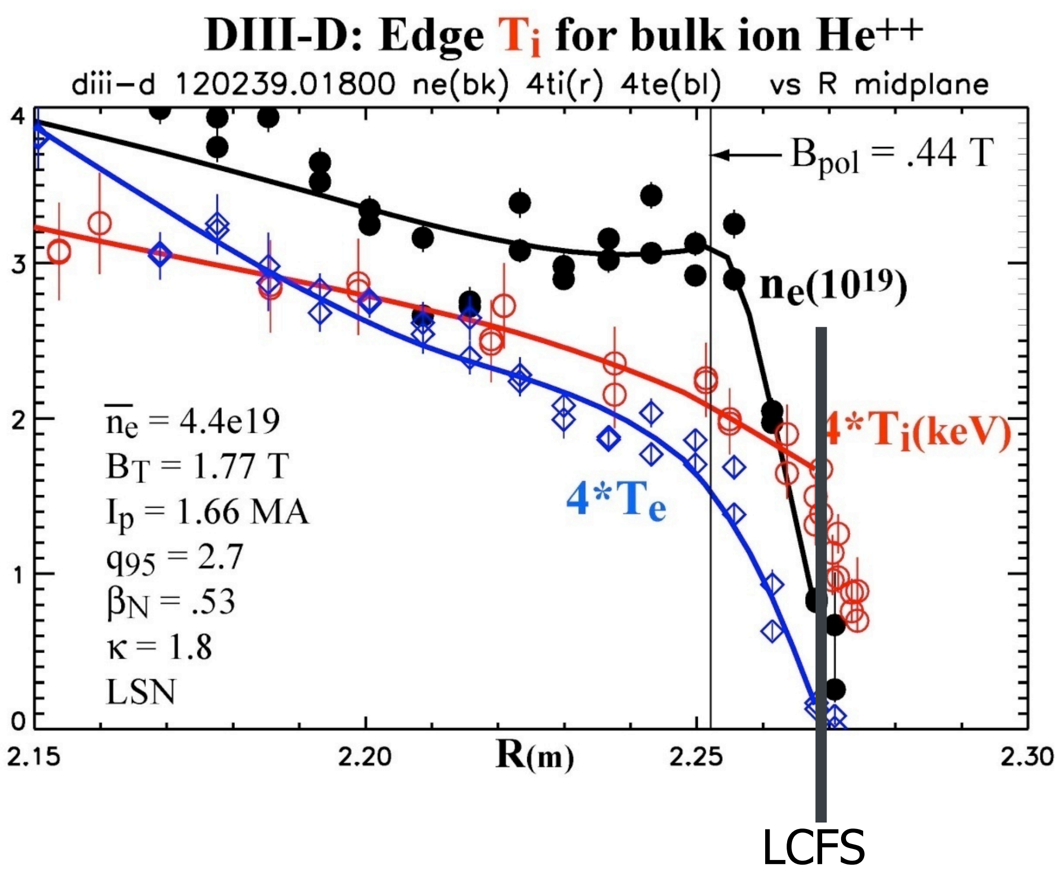
Entropy production analysis:

- The leading order ion distribution function is Maxwellian
- The ion temperature does not change over the pedestal to leading order

Physical interpretation:

- As an ion runs over a flux surface, it experiences radial excursions $\sim \rho_{pol}$, thereby equilibrating the pedestal

The T_i gradient for the bulk ion in a DIII-D ECH H-mode pedestal is small relative to gradients in electron temperature and density



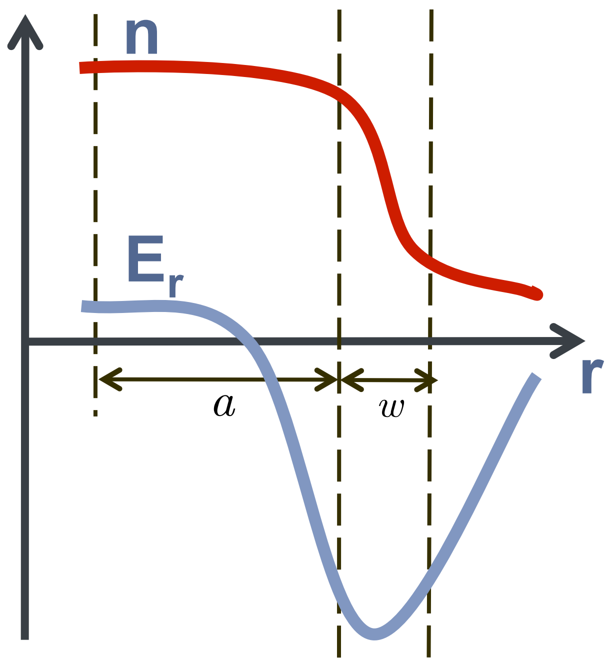
- The thermal ion full banana width is computed to be $2\rho_\theta$ = 10 mm for He^{++} at the top of the density pedestal.
- The smooth spline fits to the data (solid lines) end at the LCFS as computed by EFIT. Note the clear break in slope for T_i beyond the LCFS.
- In a nominally identical companion discharge we measured T_i for the minor C^{6+} impurity constituent. The T_i profile for C^{6+} has a very similar slope to that for He^{++}, but is ~ 150 eV greater in this region, probably because this discharge had an increase in β_N of ~ 10% compared with the one shown here.

DIII-D NATIONAL FUSION FACILITY

c.f. deGrassie, Groebner, and Burrell, PoP vol 13, 112507-1 (2006).

Pedestal basics. Main ion pressure balance

$$\rho \ll \rho_{pol} \sim w \ll a$$



Rigid toroidal rotation $\left(\vec{V}_i = \omega_i R^2 \nabla\zeta\right)$:

$$\omega_i = -c\frac{d\phi}{d\psi} - \frac{cT_i}{en}\frac{dn}{d\psi}$$

$$\omega_i \Big/ \frac{cT_i}{en}\frac{dn}{d\psi} \sim \frac{\omega_i R}{v_i} \ll 1$$

Diamagnetic and ExB drifts cancel each other to leading order

The key feature of a subsonic pedestal directly affecting particle orbits is the strong radial electric field needed to sustain ion pressure balance.

Impurity flow measurements at Alcator C-Mod:

Observation: In the banana regime pedestal, the boron impurity poloidal flow is larger than that predicted by conventional formulas.

These impurities are highly collisional and their mean free path is too short for their kinetic energy to be affected by the electric field.

It may only be the friction with background ions that the electric field manifests itself through

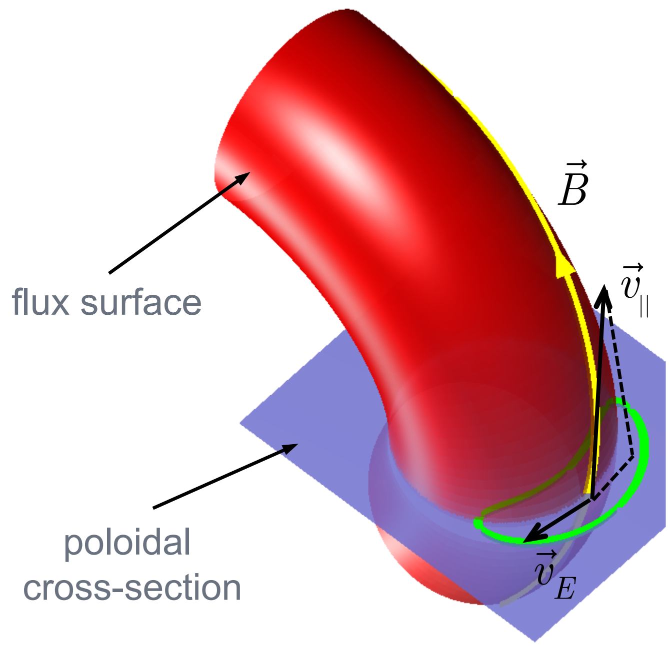
Relation between the background and impurity poloidal ion flows:

$$V_z^{pol} = V_i^{pol} - \frac{cIB_{pol}}{eB^2}\left(\frac{1}{n_i}\frac{\partial p_i}{\partial \psi} - \frac{1}{Zn_z}\frac{\partial p_z}{\partial \psi}\right)$$

If V_i^{pol} goes smaller or even negative it no longer competes with the diamagnetic terms, thereby resulting in a relatively large V_z^{pol}. Such a change in V_i^{pol} should alter the bootstrap current as well.

Main ion orbits in pedestal

$$\dot{\theta} \approx \left[v_{\parallel} + cI\phi'(\psi)/B \right] \hat{n} \cdot \nabla\theta$$



ExB drift is of order $v_{th}(\rho/\rho_{\mathrm{pol}}) << v_{||}$, but due to the geometrical factors its contribution to the poloidal velocity is comparable to that of $v_{||}$

Equations of motion

Assume a quadratic potential well and expand about ψ_*

$$\phi = \alpha + \phi_*'\left(\psi - \psi_*\right) + \frac{\phi_*''}{2}\left(\psi - \psi_*\right)^2$$

then, using μ and ψ_* invariance we can write energy conservation as

$$\frac{\left[\dot{\theta} \,/\, \left(\hat{n} \cdot \nabla\theta\right)\right]^2}{2S} - \frac{\left(cI\phi_*' \,/\, B\right)^2}{2S} + \mu B = const.$$

Contribution from ExB

magnetic dipole energy

$S \equiv 1 + cI^2\phi''/B\Omega$
orbit squeezing

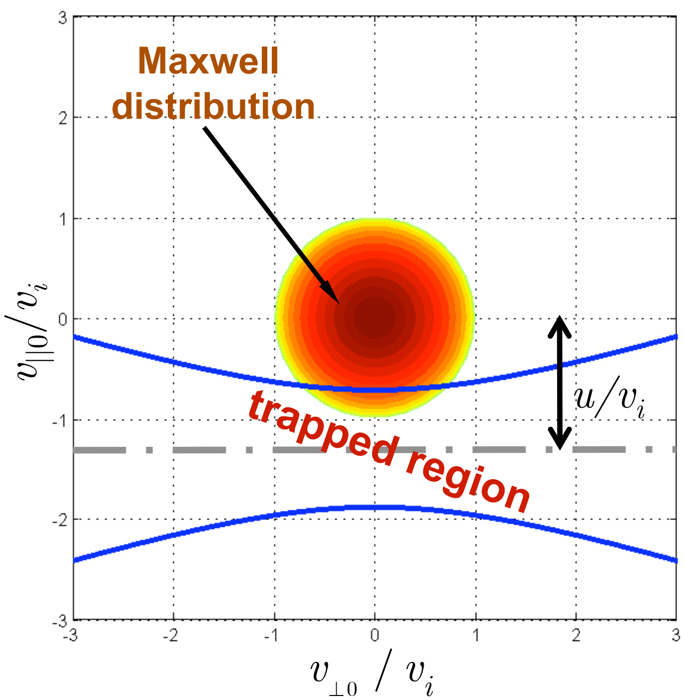
Trapped-passing boundary:

$$\left[\left(v_{\|0}\right)_{\max} + u\right]^2 \approx 4\varepsilon S\left[\mu B_0 + u^2\right]$$, where $u \equiv cI\phi_*'/B_0 \approx \left(\rho_{pol} \,/\, \rho\right)v_E$

the subscript "0" corresponds to the outboard equatorial plane (θ=0)

TRAPPED PARTICLE REGION

$$\left[\left(v_{\|0}\right)_{\max} + u\right]^2 \approx 4\varepsilon\left[v_{\perp 0}^2 / 2 + u^2\right]$$



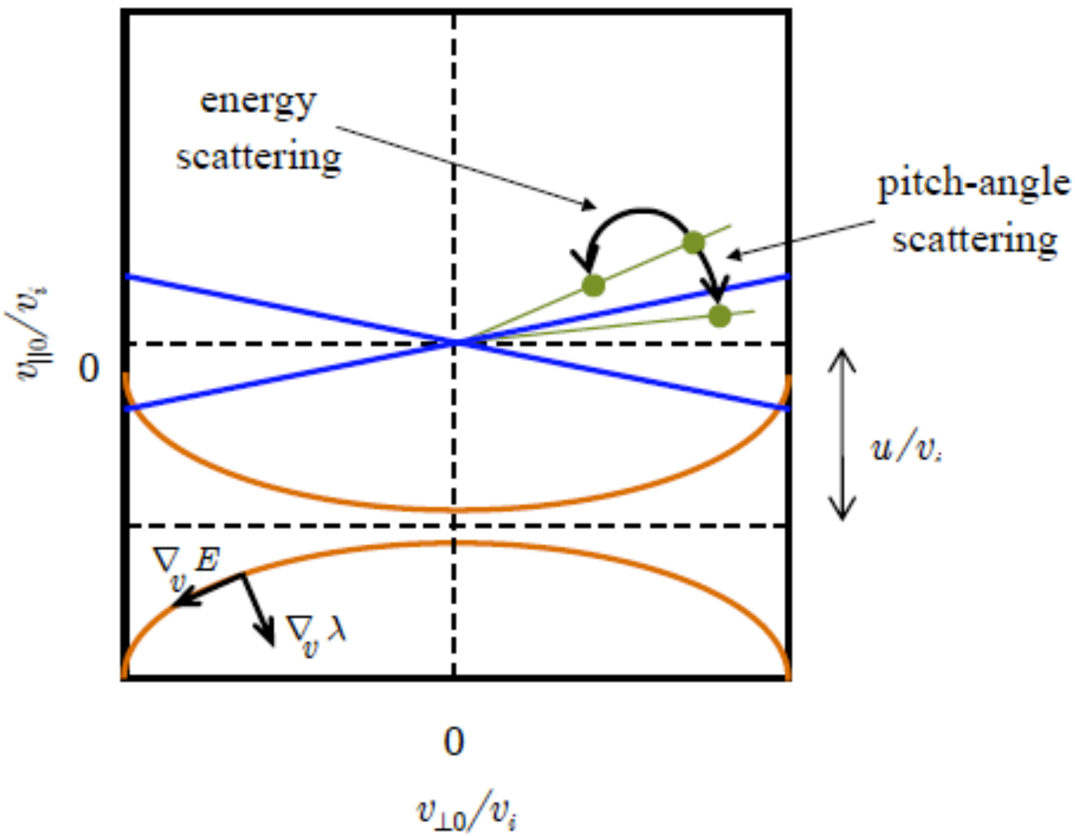
In the absence of orbit squeezing (S=1), ExB drift has the following effects:

1) Increases the depth of the effective potential well – now particles with no magnetic moment can be trapped.
2) Shifts the axis of symmetry of the trapped particles region.

For small enough ε the trapped particle fraction decays exponentially as |u| grows. Accordingly, neoclassical ion heat flux and polarization disappear in the large electric field limit. Consequence: ZF residual tends to 1.

Notice, that $u \approx (\rho_{pol}/\rho) v_E \gg v_E$ and therefore particle dynamics can be qualitatively changed even by the ExB drift much less than v_i.

NEED FOR A NEW MODEL COLLISION OPERATOR IN THE PEDESTAL



In the pedestal, the pitch angle scattering component of the collision operator is not sufficient to retain transitions across the trapped-passing boundary!

NEW VARIABLES FOR THE COLLISION OPERATOR

$$E \equiv \frac{\left(v_{||} + u\right)^2}{2S} + \frac{B}{B_0}\left(\mu B_0 + u^2\right) \qquad \lambda \equiv \frac{\left(\mu B_0 + u^2\right)}{E}$$

1) In the absence of the electric field (u=0), reduce to the conventional variables, $v^2/2$ and $2\mu B_0/v^2$

2) Almost orthogonal near the trapped-passing boundary so that only $\partial/\partial\lambda$ contribute to neoclassical processes

THE PEDESTAL MODEL FOR THE LIKE PARTICLE COLLISION OPERATOR

Start from the Rosenbluth form :

$$C_R\left\{\delta f\right\} = \nabla_v \cdot \vec{\Gamma}\left\{\delta f\right\}$$

$$\vec{\Gamma}\left\{\delta f\right\} \equiv \gamma f_0 \nabla_v \nabla_v G_M \cdot \nabla_v\left(\delta f / f_0\right)$$

$$\gamma \nabla_v \nabla_v G_M = \frac{\nu_\perp}{4}\left(v^2 \overleftrightarrow{I} - \vec{v}\vec{v}\right) + \frac{\nu_{||}}{2}\vec{v}\vec{v}$$

where

$$\nu_\perp \equiv \frac{3\sqrt{2\pi}}{2x^3}\left[\operatorname{erf}\left(x\right) - \Psi\left(x\right)\right]\nu_B$$

$$\nu_{||} \equiv \frac{3\sqrt{2\pi}}{2x^3}\Psi\left(x\right)\nu_B$$

$$\nu_B = 4\pi^{1/2} Z^4 e^4 n_i \ln\Lambda / 3M^{1/2} T^{3/2}$$

$$\Psi\left(x\right) \equiv \frac{\operatorname{erf}\left(x\right) - x \operatorname{erf}'\left(x\right)}{2x^2}$$

Switching to new variables:

$$C_{ped}\left\{\delta f\right\} = \frac{B_0\left(v_{||} + u\right)}{BE}\frac{\partial}{\partial\lambda}\left[\frac{BE}{B_0\left(v_{||} + u\right)}\vec{\Gamma}\cdot\nabla_v\lambda\right]$$

$$\vec{\Gamma}\left\{\delta f\right\}\cdot\nabla_v\lambda = f_0 \frac{\lambda\left(v_{||} + u\right)^2 B_0}{S^2 E^2 B}\left[\frac{\nu_\perp}{4}v^2 + \left(\frac{\nu_{||}}{2} - \frac{\nu_\perp}{4}\right)u^2\lambda\right]\frac{\partial}{\partial\lambda}\left[\frac{\delta f}{f_0}\right]$$

Momentum conservation

The new model collision operator previously defined does not manifestly conserve momentum. To address this issue introduce a free parameter σ to redefine

$$\vec{\Gamma}\left\{g-h\right\}\cdot\nabla_v\lambda = f_0\frac{\lambda\left(v_{\parallel}+u\right)^2 B_0}{S^2E^2B}\left[\frac{\nu_{\perp}}{4}v^2+\left(\frac{\nu_{\parallel}}{2}-\frac{\nu_{\perp}}{4}\right)u^2\lambda\right]\frac{\partial}{\partial\lambda}\left[\frac{g-h}{f_0}-\frac{\sigma I\left(v_{\parallel}+u\right)}{\Omega T}\frac{\partial T}{\partial\psi}\right]$$

Then, after solving for the first order correction to the distribution function we can find σ such that our operator conserves momentum.

Notice, it is the momentum exchange between the trapped/barely-passing and freely-passing fractions that plays the key role in establishing the ion flow

Passing constraint in the pedestal

$$\oint_* C_{ii}^l \left\{ g - \frac{Iv_{\parallel}}{\Omega} f_M \frac{Mv^2}{2T^2} \frac{\partial T}{\partial \psi} \right\} d\theta / \dot{\theta} = 0$$

where g stands for the non-diamagnetic perturbation of the Maxwellian f_M

with the integration to be completed holding energy, magnetic moment and canonical angular momentum (ψ_*) fixed. Also

$$\dot{\theta} \approx \left[v_{\parallel} + cI\phi'\left(\psi\right) / B \right] \hat{n} \cdot \nabla\theta \equiv \left(v_{\parallel} + u \right) \hat{n} \cdot \nabla\theta$$

to account for the effect of the ExB drift.

Then, using number, energy and momentum conservations we rewrite the above constraint

$$\oint_* \frac{d\theta B}{\left(v_{\parallel} + u\right) \vec{B} \cdot \nabla\theta} C\left\{ g - h \right\} = 0$$

where $h \equiv f_0 \frac{I\left(v_{\parallel} + u\right)}{\Omega} \frac{M\left(v^2 + u^2\right)}{2T^2} \frac{\partial T}{\partial \psi}$

Solution for the passing constraint

$$\frac{\partial}{\partial\lambda}\left(\frac{g}{f_0}\right)\bigg|_p \approx -\frac{ISME\left(E-\sigma T/M\right)}{\left\langle v_{\|}+u\right\rangle \Omega_0 T^2}\frac{\partial T}{\partial\psi}$$

$$\sigma=\frac{\int_0^\infty dy e^{-y}\left(y+Mu^2/T\right)^{3/2}\left[\left(y+Mu^2/2T\right)\nu_\perp+\left(Mu^2/2T\right)\left(2\nu_{\|}-\nu_\perp\right)\right]}{\int_0^\infty dy e^{-y}\left(y+Mu^2/T\right)^{1/2}\left[\left(y+Mu^2/2T\right)\nu_\perp+\left(Mu^2/2T\right)\left(2\nu_{\|}-\nu_\perp\right)\right]}$$

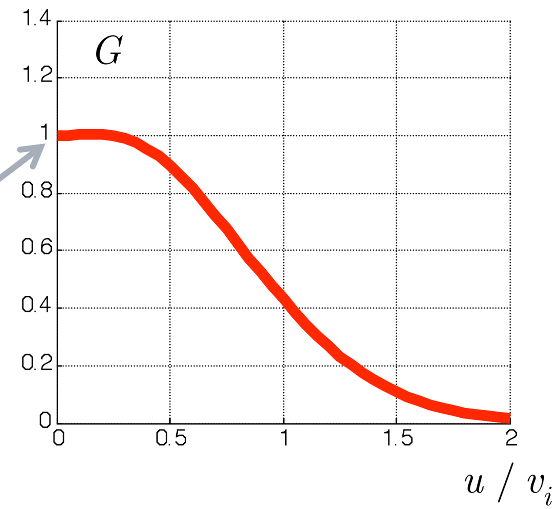
And we are in a position to calculate the neoclassical quantities of interest by taking moments of this solution.

NEOCLASSICAL ION HEAT FLUX

Moment approach: $$\left\langle \vec{q}\cdot\nabla\psi \right\rangle = -\frac{McIT}{Ze}\left\langle \int \frac{d^3v}{B}\left(\frac{Mv^2}{2T}-\frac{5}{2}\right)v_{\|}C\left\{g-h\right\}\right\rangle$$

Evaluating the integrals we find $\left\langle \vec{q}\cdot\nabla\psi \right\rangle = 1.35 n_i \nu_B \frac{TI^2\sqrt{\varepsilon S}}{\Omega_0^2 M}\frac{\partial T}{\partial \psi} G\left(u\right)$,

where



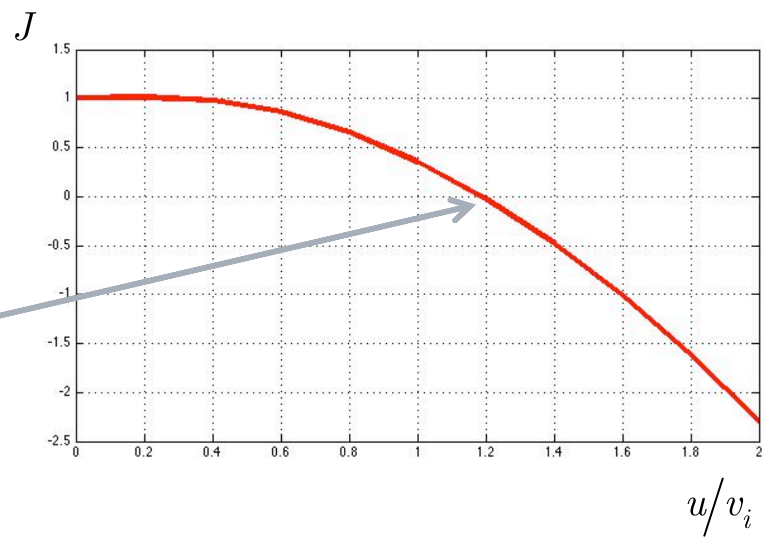
conventional result

NEOCLASSICAL ION PARALLEL FLOW

Similarly to the ion heat flux we calculate the parallel ion flow to obtain

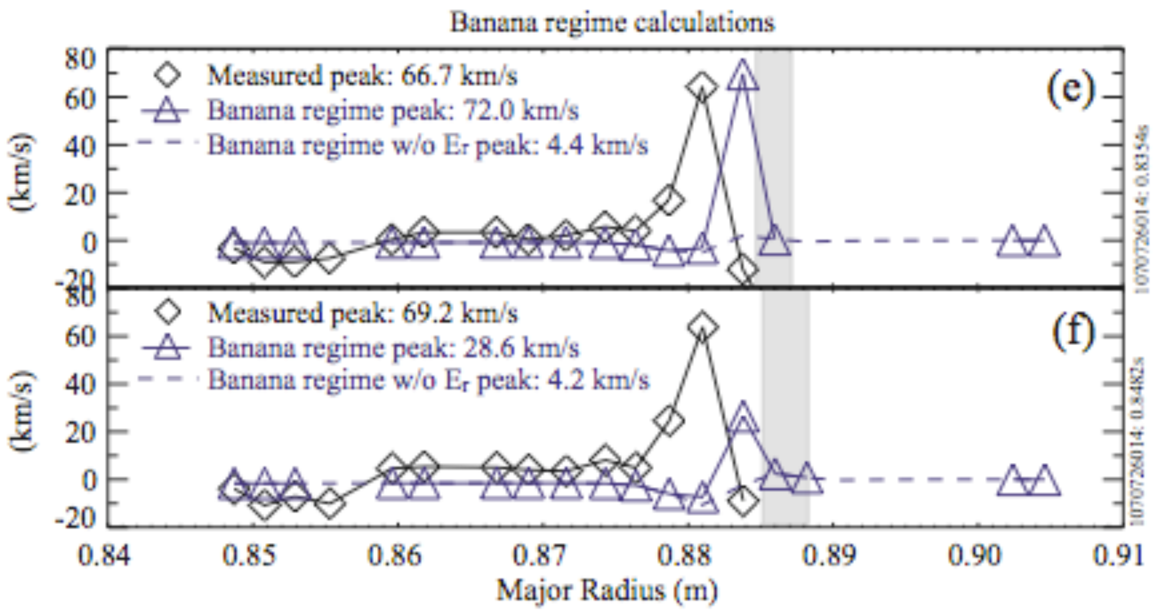
$$V_{i||} = -\frac{cI}{B}\left(\frac{\partial\phi}{\partial\psi} + \frac{1}{Zen_i}\frac{\partial p}{\partial\psi}\right) - 1.17\frac{I}{\Omega_0 M}\frac{\partial T}{\partial\psi}J\left(u\right)$$

poloidal flow



Poloidal flow changes direction at $u/v_i \approx 1.2$!

Impurity measurements at C-Mod yet again



Courtesy of K.D. Marr

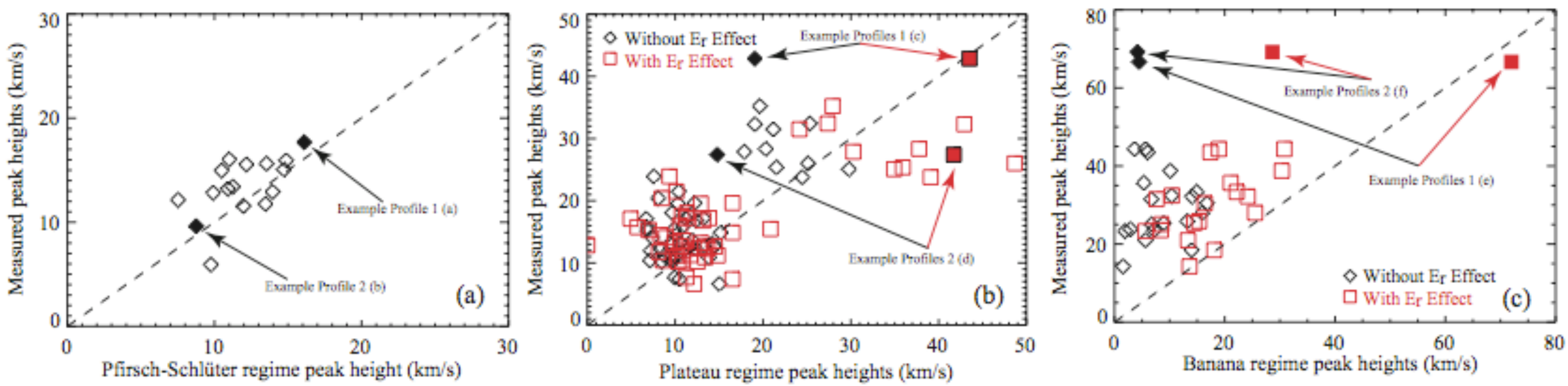
good match

bad match

Typical discrepancy: peaks in measured and predicted V_z are misaligned, but the former is aligned with the peak in E_r. Since direct measurements of T_z are not available T_i=T_Z is set. Recall $u/v_i \propto E_r/\sqrt{T_i}$

$T_i = T_Z$?

IMPURITY MEASUREMENTS AT C-MOD YET AGAIN



- For the PS regime, conventional approach does fairly well
- For the plateau regime, it is not clear which approach is better
- For the banana regime, the agreement is noticeably improved if E_r effects are accounted for

ENHANCEMENT OF THE BOOTSTRAP CURRENT IN THE BANANA REGIME PEDESTAL

The calculation of the bootstrap current runs the same as in the conventional case with the new expression for the parallel ion flow in place of the old one

$$J_{BS} \approx -1.46\varepsilon^{1/2}\frac{cIB}{\left\langle B^2\right\rangle}\left[\frac{Z^2+2.21Z+0.75}{Z\left(Z+1.414\right)}\right]\left[\frac{dp}{d\psi}-\frac{\left(2.07Z+0.88\right)n_e}{Z^2+2.21Z+0.75}\frac{dT_e}{d\psi}-1.17J\left(u\right)\frac{n_e}{Z_i}\frac{dT_i}{d\psi}\right]$$

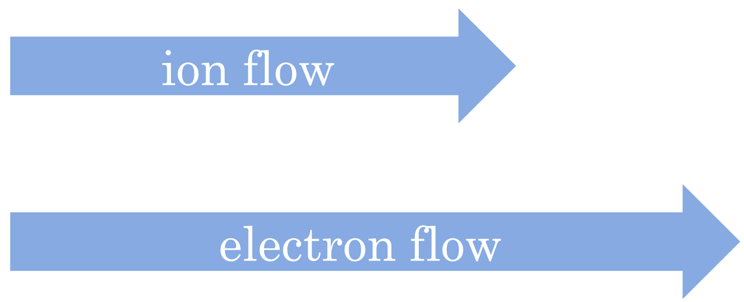
$$Z\to\infty: \quad J_{BS} \approx -1.46\varepsilon^{1/2}\frac{cIB}{\left\langle B^2\right\rangle}\left[\frac{dp}{d\psi}-1.17J\left(u\right)n_i\frac{dT_i}{d\psi}\right]$$

governed by poloidal ion flow

Analogously to the impurity case, poloidal ion flow going negative results in a bootstrap current larger than that predicted by the conventional theory

PHYSICAL PICTURE

Conventional case:



Electric field acts to increase the difference between the electron and ion flow.
The friction grows in response, but the bootstrap current is still enhanced.

SUMMARY

- In the banana regime pedestal ion orbits are modified by a strong radial electric field, thereby calling for reexamination of conventional neoclassical theory

- In particular, neoclassical poloidal ion flow changes direction as compared to its core counterpart
  - This result is confirmed by impurity measurements at the Alcator C-Mod
  - Consequently, the bootstrap current in the pedestal is larger that one might expect based on conventional consideration